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EXPECTED VALUE OF THE SAMPLE MEAN

I. Objective:

We would like to show that the sample mean, $\bar{X}$, is an unbiased estimator of the population mean, μ. That is:

$$E(\bar{X}) = \mu$$

In order to accomplish this, we must calculate the expected value of the sample mean.

II. The Expected Value of the Sample Mean:

Our strategy is to start with the definition of the sample mean, recognize that it is a linear combination of the data values, and then employ the properties of summations, and expected values in order to reach the desired result:

$$\bar{X} = \frac{\sum X_i}{n}$$

$$= \frac{X_1}{n} + \frac{X_2}{n} + \ldots + \frac{X_n}{n}$$

$$= \frac{1}{n} X_1 + \frac{1}{n} X_2 + \ldots + \frac{1}{n} X_n$$

$$E(\bar{X}) = E(\frac{1}{n} X_1 + \frac{1}{n} X_2 + \ldots + \frac{1}{n} X_n)$$

$$= E(\frac{1}{n} X_1) + E(\frac{1}{n} X_2) + \ldots + E(\frac{1}{n} X_n)$$

$$= \frac{1}{n} E(X_1) + \frac{1}{n} E(X_2) + \ldots + \frac{1}{n} E(X_n)$$

$$= \frac{1}{n} \left[E(X_1) + E(X_2) + \ldots + E(X_n)\right]$$

$$= \frac{1}{n} [\mu + \mu + \ldots + \mu]$$

$$= \frac{1}{n} [n \mu]$$

$$E(\bar{X}) = \mu$$

Thus, the sample mean is an unbiased estimator of the population mean.